Reciprocal and Pythagorean Identities

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A little algebra review. . .

Factor:

1. $x^3 + x$

$x(x^2+1)$

2. $x^2 + 9x + 14$

$(x+7)(x+2)$

3. $9y^2 - 16$

$(3y+4)(3y-4)$

4. ~~$\sin 4\theta + \sin 2\theta$~~

$\sin^4\theta + \sin^2\theta$

$\sin^2\theta(\sin^2\theta + 1)$

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Factor these, too

5. $4-25\cos^2 x$

$(2+5\cos x)(2-5\cos x)$

6. $\sin^2\theta-2\sin\theta$

$\sin\theta(\sin\theta-2)$

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Simplify

b. $\frac{4}{a}+\frac{2}{b}$ a / $b\cdot a$

$\frac{4b}{ab}+\frac{2a}{ab}$

$\frac{4b+2a}{ab}$

$(x+5)\frac{1}{(x-5)(x+3)}+\frac{4(x+3)}{x-5(x+3)}$

$\frac{x+5}{(x+5)(x+3)}+\frac{4(x+3)}{(x+3)(x-5)}$

$\frac{x-5+4x+12}{(x+3)(x-5)}$

$\frac{5x+7}{(x+3)(x-5)}$

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Simplify

$$\frac{1}{\sin\theta}+\frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$\frac{\cos\theta}{\cos\theta\sin\theta}+\frac{\sin^2\theta}{\cos\theta\sin\theta}$

$\frac{\cos\theta+\sin^2\theta}{\cos\theta\sin\theta}$

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Fundamental Identities

Reciprocal identities

$\sin x = \frac{1}{\csc x}$ $\quad \csc x = \frac{1}{\sin x}$

$\cos x = \frac{1}{\sec x}$ $\quad \sec x = \frac{1}{\cos x}$

$\tan x = \frac{1}{\cot x}$ $\quad \cot x = \frac{1}{\tan x}$

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II. Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

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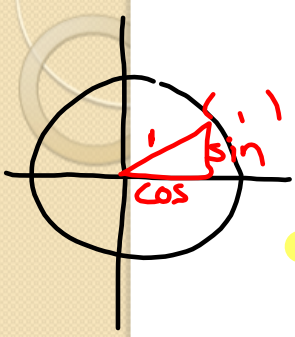
Pythagorean Identities



$$\sin^2 x + \cos^2 x = 1$$

$-\cos^2 x \quad -\cos^2 x$

$$\sin^2 x = 1 - \cos^2 x$$

$\sin^2 + \cos^2 = 1$

$-\sin^2 \quad -\sin^2$

$$\cos^2 = 1 - \sin^2$$

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More Pythags

$$\frac{\sin^2 x}{\sin^2} + \frac{\cos^2 x}{\sin^2} = \frac{1}{\sin^2}$$

$1 + \cot^2 = \csc^2$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$\tan^2 + 1 = \sec^2$

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IV. Even/Odd Identities

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Simplify

$\sin x \frac{1}{\cos x}$

① Look for an identity

$\frac{\sin x}{\cos x}$

② Change Sin/cos

$\tan x$

③ Combine any fractions

④ GCF Simplify

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Simplify

$\sin x \frac{1}{\sin x} - \sin x \frac{1}{\sin x}$

$\frac{\sin x}{\sin x} - 1$

$1 - 1$

0

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Guidelines for Verifying

- **Work on one side at a time**-start with the most complicated
- **Look for places to use fundamental identities**
- **Try these techniques:**
  - a. Rewrite as sines and cosines
  - b. Factor out a GCF
  - c. Separate a single term quotient into 2 terms
  - d. Combine fractions with a common denominator
- **Don't be afraid to start over another way!**

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Verify

$$\frac{\cos\theta}{1-\cos^2\theta} = \csc\theta\cot\theta$$

$$\frac{\cos\theta}{\sin^2\theta} = \csc\theta\cot\theta$$

$$\frac{1}{\sin}\cdot\frac{\cos}{\sin}$$

$$\csc\theta\cot\theta = \csc\theta\cot\theta$$

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Verify:

$$1-\cos\theta\sin\theta\cot\theta=\sin^2\theta$$

$1-\cos\theta\sin\theta\dfrac{\cos\theta}{\sin\theta}$

$1-\cos^2\theta$

$\sin^2\theta$

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Verify:

$$\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=2\sec^2\theta$$

$\dfrac{(1+\sin\theta)}{(1+\sin\theta)}\cdot\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}\cdot\dfrac{(1-\sin\theta)}{(1-\sin\theta)}$

$\dfrac{1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}+\dfrac{1-\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$\dfrac{2}{(1+\sin\theta)(1-\sin\theta)}$

$\dfrac{2}{(1-\sin^2\theta)}$

$\dfrac{2}{\cos^2}$

$2\cdot\dfrac{1}{\cos^2}$

$2\sec^2$ ✓

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